III Semester M.Sc. Degree Examination, December 2014
(Semester Scheme) (NS)
MATHEMATICS
M-301 : Topology – II

Time : 3 Hours Max. Marks : 80

Instructions : *i) Answer **any five full** questions choosing atleast **two** from **each** Part.*
*ii) **All** questions carry **equal** marks.*

PART – A

1. a) Prove that a topological space is compact if and only if every family of closed sets having the finite intersection property has a non-empty intersection. **8**

   b) Show that a metric space (x, d) is sequentially compact if and only if it is countably compact. **8**

2. a) Define a locally compact space. Prove that a T_2 -space is locally compact if and only if each point has a neighbourhood whose closure is compact. **6**

   b) Prove that a compact metric space is complete. **5**

   c) Show that a complete totally bounded metric space is compact. **5**

3. a) Define :

      i) Second countable space

      ii) Separable space.

   Show that every second countable space is a separable space. Is the converse true ? Justify. **6**

   b) Show that the property of being a separable space is topological. **4**

   c) Show that every second countable space is Lindelof. **6**

4. a) Show that in a T_0 - space, the closures of distinct points are distinct. **6**

b) Show that a topological space is a T_1 - space if and only if all singletons in it are closed. **5**

c) What is T_2 - space ? Prove that in a T_2 - space a convergent sequence has a unique limit. **5**

PART – B

5. a) Define a T_3 - space, prove that a topological space (X, T) is regular if and only if given an open set G and $x \in G$ there exists an open set G* such that $x \in G^* \subset \overline{G^*} \subset G$. **6**

b) Prove that the property of being a normal space is closed hereditary. **4**

c) Prove that a compact Hausdorff space is normal. **6**

6. a) State and prove Urysohn's lemma. **10**

b) Prove that a topological space X is completely regular if and only if for every $x \in X$ and every open set G containing x there exists a continuous mapping f of X into [0, 1] such that f(x) = 0 and f(y) = 1 $\forall y \in X - G$. Hence prove that for every two distinct points x and y in a Tychnoff space X there exists a real valued continuous mapping f of X such that f(x) $\neq$ f(y). **6**

7. a) Show that a normal space is completely regular if and only if it is regular. **6**

b) State and prove Alexandroff's one point compactification. **10**

8. a) Define : **8**

i) α-cuts

ii) strong α-cuts

Let X be a set and A be a fuzzy subset of X then prove that

i) if $\alpha \leq \beta$ then $\alpha_A \supset \beta_A$

ii) if $\alpha \leq \beta$ then $\alpha_{+A} \supset \beta_{+A}$ for $\alpha, \beta \in [0, 1]$.

b) State and prove first decomposition theorem. **8**